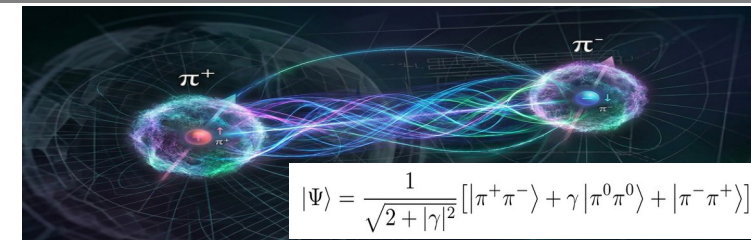


Quantum Entangled Pions and implications for CP violations

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Works in collaboration with: M. Fabbrichesi, R. Floreanini, L. Marzola

Fabbrichesi, Floreanini, EG, Marzola, PRD 112 (2025) 9, 9; [arXiv:2506.05464](#)
PRD 112 (2025) 3, 033005; [arXiv:2504.12382](#)
[arXiv:2512.07939](#)

Summary

- Particles produced in scattering and decay can become entangled in spin states
- similarly, isospin in flavor space can also become entangled
- pions carry isospin $I = 1$ and π^+ π^- π^0 are members of an isospin triplet
- Main idea: analogy with the case of spin-1 particles but for internal isospin SU(2)
- study the quantum properties of pion iso-triplet from two- and three-pion final states from CP conserving decays of K_S and K_L
- quantum states reconstructed by using quantum state tomography in flavor space
- we will analyze: entanglement, Bell non-locality, and contextuality
- New proposal: use of trace distance of density matrices (in flavor space) to test direct CP violations in Kaon, D- and B-mesons decays

Two-pion states

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Quantum-entangled pions*, *Phys. Rev. D* **112** (2025), no. 9 L091301, [arXiv:2506.05464].

- Quantum state $|\Psi\rangle$ of two pions emerging from the Kaon decay in its rest frame :

$$K^0 \rightarrow \pi\pi$$

Charge Isospin (3rd component)

- Configuration of the state determined by the condition $\rightarrow Q = I_3 = 0$

$$|\Psi\rangle = \frac{1}{\sqrt{2 + |\gamma|^2}} [|\pi^+\pi^-\rangle + \gamma |\pi^0\pi^0\rangle + |\pi^-\pi^+\rangle]$$

Density matrix

$$\rho = |\Psi\rangle\langle\Psi|$$



$$\rho = \frac{1}{2 + |\gamma|^2}$$

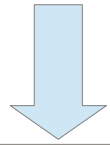
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \gamma^* & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 & |\gamma|^2 & 0 & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \gamma^* & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

on the basis

$$\pi^+ = (1, 0, 0)^T, \pi^0 = (0, 1, 0)^T \text{ and } \pi^- = (0, 0, 1)^T$$

- Neglecting **CP** violation (good approximation) → set CP phases to zero
- we determine the parameter **γ** from isospin **$I=0,2$** amplitudes $A_I = \langle \pi\pi(I) | K^0 \rangle$

$$\begin{aligned}\langle \pi^+ \pi^- | K^0 \rangle &= A_0 e^{i\delta_0} + \frac{A_2}{\sqrt{2}} e^{i\delta_2} \\ \langle \pi^0 \pi^0 | K^0 \rangle &= A_0 e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2}\end{aligned}$$

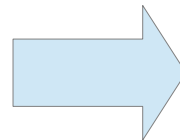


$$\gamma = \frac{\langle \pi^0 \pi^0 | K^0 \rangle}{\langle \pi^+ \pi^- | K^0 \rangle} = \frac{\sqrt{2} A_0 e^{i(\delta_0 - \delta_2)} - 2 A_2}{\sqrt{2} A_0 e^{i(\delta_0 - \delta_2)} + A_2}$$

$$A_0 = (2.704 \pm 0.001) \times 10^{-7} \text{ GeV}$$

$$A_2 = (1.210 \pm 0.002) \times 10^{-8} \text{ GeV}$$

$$\delta_0(m_K) - \delta_2(m_K) = (47.5 \pm 0.9)^\circ,$$



$$|\gamma|^2 = 0.880 \pm 0.002$$

Entanglement and Bell non-locality

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Quantum-entangled pions*, *Phys. Rev. D* **112** (2025), no. 9 L091301, [[arXiv:2506.05464](#)].

- Pions are in pure states so entanglement can be quantified by **Entropy of entanglement**

$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

$$0 \leq \mathcal{E}[\rho] \leq \ln d$$

d=2 for qubits
d=3 for qutrits

$$\mathcal{E}[\rho] = \frac{1}{|\gamma|^2 + 2} \left[|\gamma|^2 \log \left(\frac{|\gamma|^2 + 2}{|\gamma|^2} \right) + 2 \log (|\gamma|^2 + 2) \right]$$

$$\mathcal{E}[\rho] = 1.097 \pm 0.001$$

- Bell locality criteria** for two qutrits $\mathcal{I}_3 \leq 2$ **Collins, Gisin, Linden, Massar, Popescu, PRL 88 (2002)**

$$\mathcal{I}_3 = \text{Tr} [\mathcal{B} \rho]$$



$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$ to maximize inequality



$$\mathcal{I}_3 = 2.890 \pm 0.001$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

- Two pions from K^0 decays are both entangled and non-locals!**

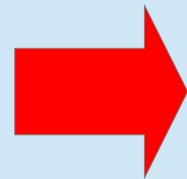
Contextuality

- impossibility of assigning a value to the result of a measurement of a physical property independently of the measurement context or assigning simultaneous values to all observables of QM system
- **Bell-Kochen-Specker theorem**: context-independent classical description of the predictions of QM is indeed impossible
- Smallest Hilbert space in which contextuality can be manifest is 3 → one **qutrit**

$$[A, B] = 0$$

$$[B, C] \neq 0$$

$$[A, C] = 0$$



$$(A \ B)$$

$$\neq$$

$$(A \ C)$$

J. S. Bell, *On the Problem of Hidden Variables in Quantum Mechanics*, Rev. Mod. Phys. **38** (1966) 447–452.

S. Kochen and E. Specker, *The Problem of Hidden Variables in Quantum Mechanics*, J. Math. Mech. **17** (1968) 59–87.

→ **Bell-Kochen-Specker theorem**

Experimental tests of contextuality

photons

M. Michler, H. Weinfurter, and M. Żukowski, *Experiments towards falsification of noncontextual hidden variable theories*, Phys. Rev. Lett. **84** (Jun, 2000) 5457–5461.

Y.-F. Huang, C.-F. Li, Y.-S. Zhang, J.-W. Pan, and G.-C. Guo, *Experimental test of the Kochen-Specker theorem with single photons*, Phys. Rev. Lett. **90** (Jun, 2003) 250401.

E. Amselem, M. Rådmark, M. Bourennane, and A. Cabello, *State-independent quantum contextuality with single photons*, Phys. Rev. Lett. **103** (Oct, 2009) 160405.

R. Lapkiewicz, P. Li, C. Schaeff, N. K. Langford, S. Ramelow, M. Wieśniak, and A. Zeilinger, *Experimental non-classicality of an indivisible quantum system*, Nature **474** (2011), no. 7352 490–493.

C. Zu, Y.-X. Wang, D.-L. Deng, X.-Y. Chang, K. Liu, P.-Y. Hou, H.-X. Yang, and L.-M. Duan, *State-independent experimental test of quantum contextuality in an indivisible system*, Phys. Rev. Lett. **109** (Oct, 2012) 150401.

V. D'Ambrosio, I. Herbauts, E. Amselem, E. Nagali, M. Bourennane, F. Sciarrino, and A. Cabello, *Experimental implementation of a kochen-specker set of quantum tests*, Phys. Rev. X **3** (Feb, 2013) 011012.

J. Ahrens, E. Amselem, A. Cabello, and M. Bourennane, *Two fundamental experimental tests of nonclassicality with qutrits*, Scientific Reports **3** (2013), no. 1 2170.

ions

G. Kirchmair, F. Zähringer, R. Gerritsma, M. Kleinmann, O. Gühne, A. Cabello, R. Blatt, and C. F. Roos, *State-independent experimental test of quantum contextuality*, Nature **460** (2009), no. 7254 494–497.

X. Zhang, M. Um, J. Zhang, S. An, Y. Wang, D.-l. Deng, C. Shen, L.-M. Duan, and K. Kim, *State-independent experimental test of quantum contextuality with a single trapped ion*, Phys. Rev. Lett. **110** (Feb, 2013) 070401.

F. M. Leupold, M. Malinowski, C. Zhang, V. Negnevitsky, A. Cabello, J. Alonso, and J. P. Home, *Sustained state-independent quantum contextual correlations from a single ion*, Phys. Rev. Lett. **120** (May, 2018) 180401.

molecular nuclear spin

O. Moussa, C. A. Ryan, D. G. Cory, and R. Laflamme, *Testing contextuality on quantum ensembles with one clean qubit*, Phys. Rev. Lett. **104** (Apr, 2010) 160501.

neutrons

Y. Hasegawa, R. Loidl, M. Baron, and H. Rauch, *Violation of a bell-like inequality in single-neutron interferometry*, Nature **425** (2003) 45.

superconducting systems

M. Jerger, Y. Reshitnyk, M. Oppliger, A. Potočník, M. Mondal, A. Wallraff, K. Goodenough, S. Wehner, K. Juliusson, N. K. Langford, and A. Fedorov, *Contextuality without nonlocality in a superconducting quantum system*, Nature

nuclear spins

S. B. van Dam, J. Cramer, T. H. Taminiau, and R. Hanson, *Multipartite entanglement generation and contextuality tests using nondestructive three-qubit parity measurements*, Phys. Rev. Lett. **123** (Jul, 2019) 050401.

loopholes: sharpness, detection, compatibility

3-level systems (qutrits)

$\mathcal{O}_i, i = 1, 2, 3, 4, 5 \longrightarrow$ dichotomic observables taking values $+1, -1$

$\mathcal{O}_i$ commutes with $\mathcal{O}_{i+1}$ but in general not with remaining ones

These observables have to be measured in pairs $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)\}$

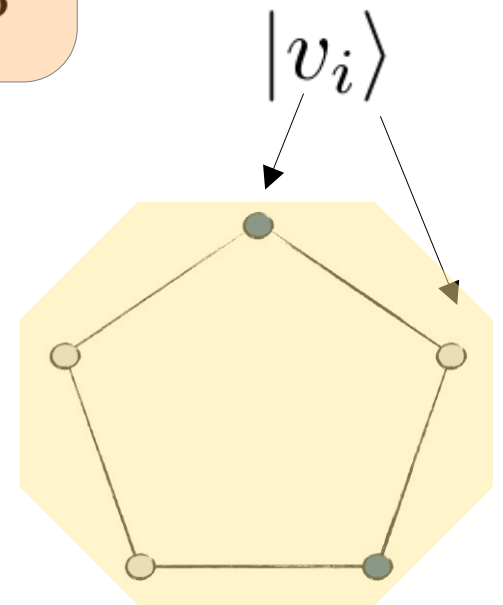
$$o_1 o_2 + o_2 o_3 + o_3 o_4 + o_4 o_5 + o_5 o_1 \geq -3$$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle + \langle \mathcal{O}_2 \mathcal{O}_3 \rangle + \langle \mathcal{O}_3 \mathcal{O}_4 \rangle + \langle \mathcal{O}_4 \mathcal{O}_5 \rangle + \langle \mathcal{O}_5 \mathcal{O}_1 \rangle \geq -3$$

Can be represented by introducing projectors $\Pi_i \equiv |v_i\rangle\langle v_i|$

vectors $|v_i\rangle, i = 1, \dots, 5$, with $\langle v_i | v_{i+1} \rangle = 0$

$$\mathcal{O}_i = 1 - 2\Pi_i \longrightarrow \sum_{i=1}^5 \langle \Pi_i \rangle \leq 2$$



3-level systems (qutrits)

Contextuality can be tested also with **9** and **13 vectors**

although with 13 vectors the test becomes independent of the state

J. Ahrens, E. Amselem, A. Cabello, and M. Bourennane, *Two fundamental experimental tests of nonclassicality with qutrits*, Scientific Reports **3** (2013), no. 1 2170.

5-vectors

$$|v_j\rangle = \left(\cos \phi, \sin \phi \cos \frac{4\pi j}{5}, \sin \phi \sin \frac{4\pi j}{5} \right)^T \quad \text{with} \quad \cos^2 \phi = \frac{\cos \pi/5}{1 + \cos \pi/5}$$

P. Kurzyński and D. Kaszlikowski, *Contextuality of almost all qutrit states can be revealed with nine observables*, Phys. Rev. A **86** (Oct, 2012) 042125.

9-vectors

$$\begin{aligned} |v_1\rangle &= (1, 0, 0)^T, & |v_2\rangle &= (0, 1, 0)^T, & |v_3\rangle &= (0, 0, 1)^T, \\ |v_4\rangle &= 1/\sqrt{2}(0, 1, -1)^T, & |v_5\rangle &= 1/\sqrt{3}(1, 0, -\sqrt{2})^T, & |v_6\rangle &= 1/\sqrt{3}(1, \sqrt{2}, 0)^T, \\ |v_7\rangle &= 1/2(\sqrt{2}, 1, 1)^T, & |v_8\rangle &= 1/2(\sqrt{2}, -1, -1)^T, & |v_9\rangle &= 1/2(\sqrt{2}, -1, 1)^T. \end{aligned}$$

$$\text{CNT}\mathbb{X}\text{T}_N \equiv \sum_{i=1}^N \langle \Pi_i \rangle = \sum_{i=1}^N \text{Tr} (\rho \Pi_i) \leq c_N ,$$

$$c_5 = 2, \quad c_9 = 3 \quad \text{and} \quad c_{13} = 4$$

Back to pions

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Quantum-entangled pions*, *Phys. Rev. D* **112** (2025), no. 9 L091301, [[arXiv:2506.05464](#)].

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Tests of quantum contextuality in particle physics*, *Phys. Rev. D* **112** (2025), no. 3 033005, [[arXiv:2504.12382](#)].

reduced density matrix for one pion state

$$\rho_1 = \frac{1}{|\gamma|^2 + 2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & |\gamma|^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \text{obtained by tracing over the degrees of freedom of the other}$$

Describe incoherent mixture of the three physical states π^+ , π^0 and π^-

$$\overline{\text{CNT}\mathbb{X}\text{T}}_9 = \sum_{k=1}^4 \lambda_k^{\rho_1} \lambda_k^{\Pi} \leq 3$$

eigenvalues of $\Pi = \begin{pmatrix} 10/3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 8/3 \end{pmatrix}$

eigenvalues ρ_1

$$\text{CNT}\mathbb{X}\text{T}_9 = \begin{cases} \frac{8|\gamma|^2 + 19}{3|\gamma|^2 + 6} & \text{if } |\gamma|^2 < 1, \\ \frac{10|\gamma|^2 + 17}{3|\gamma|^2 + 6} & \text{if } |\gamma|^2 \geq 1 \end{cases}$$

non-contextuality violated with significance well above 5σ

$$\text{CNT}\mathbb{X}\text{T}_9 = 3.014 \pm 0.001$$

Three-pion states

coming from the decay of K_L

$$|\Phi\rangle = \frac{1}{\sqrt{6 + |\Gamma|^2}} \left[|\pi^0 \pi^+ \pi^-\rangle + |\pi^+ \pi^0 \pi^-\rangle + |\pi^+ \pi^- \pi^0\rangle \right. \\ \left. + \Gamma |\pi^0 \pi^0 \pi^0\rangle + |\pi^0 \pi^- \pi^+\rangle + |\pi^- \pi^0 \pi^+\rangle + |\pi^- \pi^+ \pi^0\rangle \right]$$

density matrix $\eta = |\Phi\rangle\langle\Phi|$ satisfies $\eta^2 = \eta$

reduced density matrix on each pion state

$$\eta_0 = \eta_- = \eta_+ = \frac{1}{|\Gamma|^2 + 6} \begin{pmatrix} 2 & 0 & 0 \\ 0 & |\Gamma|^2 + 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \longrightarrow \text{obtained from tracing over the remaining DOF of two pion states}$$

$$\text{CNT}\mathbb{X}\mathbb{T}_9 = \frac{10 |\Gamma|^2 + 54}{3 (6 + |\Gamma|^2)}$$

$\Gamma \longrightarrow$ taken from experimental measurements of the relative amplitudes and it depends on the kinematics of final states (see next slides)

3-level systems: entanglement

$$\mathcal{C}_{i(jk)} = \sqrt{2 \left[1 - \text{Tr} (\eta_{(jk)}^2) \right]}$$

genuine multipartite entanglement

reduced density matrix

$$\mathcal{C}[\eta] = \min \{ \mathcal{C}_{1(23)}, \mathcal{C}_{2(13)}, \mathcal{C}_{3(12)} \}$$

$$\mathcal{C}_{1(23)} = \mathcal{C}_{3(12)} = \mathcal{C}_{2(31)} = \frac{4\sqrt{|\Gamma|^2 + 3}}{|\Gamma|^2 + 6}$$

Concurrence triangle

S. Xie and J. H. Eberly, *Triangle measure of tripartite entanglement*, Phys. Rev. Lett. **127** (Jul, 2021) 040403.
Z.-X. Jin, Y.-H. Tao, Y.-T. Gui, S.-M. Fei, X. Li-Jost, and C.-F. Qiao, *Concurrence triangle induced genuine multipartite entanglement measure*, Results Phys. **44** (2023) 106155, [arXiv:2212.07067].

$$F_3 = 4\sqrt{\frac{Q(Q - \mathcal{C}_{1(23)})(Q - \mathcal{C}_{2(13)})(Q - \mathcal{C}_{3(12)})}{3}}$$



$$Q = 6\frac{\sqrt{|\Gamma|^2 + 3}}{|\Gamma|^2 + 6}$$

$$F_3 = 16\frac{|\Gamma|^2 + 3}{(|\Gamma|^2 + 6)^2}$$

$$0 \leq F_3 \leq 1$$

$$Q = \frac{1}{2} (\mathcal{C}_{1(23)}^2 + \mathcal{C}_{2(13)}^2 + \mathcal{C}_{3(12)}^2)$$

3-level systems: Bell non-locality (3 qutrits)

$$\begin{aligned}\mathcal{B}' = & A_0 \otimes B_0 \otimes C_0 + A_0 \otimes B_0 \otimes C_1 + A_0 \otimes B_1 \otimes C_0 \\ & + A_1 \otimes B_0 \otimes C_0 - A_1 \otimes B_1 \otimes C_1 - A_0 \otimes B_1 \otimes C_1 \\ & - A_1 \otimes B_0 \otimes C_1 - A_1 \otimes B_1 \otimes C_0 ,\end{aligned}$$

with condition

$$A_k^2 = B_k^2 = C_k^2 = \mathbb{1}_3$$

Bell inequality for a tripartite system

$$\mathcal{I}'_3 = \text{Tr} [\eta \mathcal{B}'] \leq 4$$

Svetlichny inequality

G. Svetlichny, *Distinguishing three-body from two-body nonseparability by a bell-type inequality*, Phys. Rev. D **35** (May, 1987) 3066–3069.

M. Seevinck and G. Svetlichny, *Bell-type inequalities for partial separability in n-particle systems and quantum mechanical violations*, Phys. Rev. Lett. **89** (Jul, 2002) 060401.

maximization matrices within 1%

$$\begin{aligned}A_0 = -B_1 = -C_0 &= \begin{pmatrix} \frac{8}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \\ \frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \end{pmatrix} \\ A_1 = B_0 = -C_1 &= \begin{pmatrix} \frac{8}{9} & -\frac{4}{9} & \frac{1}{9} \\ -\frac{4}{9} & -\frac{7}{9} & \frac{4}{9} \\ \frac{1}{9} & \frac{4}{9} & \frac{8}{9} \end{pmatrix}\end{aligned}$$

→ Max value of QM = $4\sqrt{2}$

$$\Gamma = 1.372 \longrightarrow \text{give the maximum value}$$



Max value

$$\mathcal{I}'_3 = 4.569$$

How to determine the value of relative amplitude weight Γ

Using Chiral Perturbation Theory

$$A(K_L \rightarrow \pi^0 \pi^0 \pi^0) = 3(\alpha_1 + \alpha_3) + 3(\zeta_1 - 2\zeta_3) \left(y^2 + \frac{1}{3}x^2 \right)$$

$$\begin{aligned} A(K_L \rightarrow \pi^0 \pi^+ \pi^-) &= (\alpha_1 + \alpha_3) - (\beta_1 + \beta_2)y + (\zeta_1 - 2\zeta_3) \left(y^2 + \frac{1}{3}x^2 \right) \\ &+ (\xi_1 - 2\xi_3) \left(y^2 - \frac{1}{3}x^2 \right) \end{aligned}$$

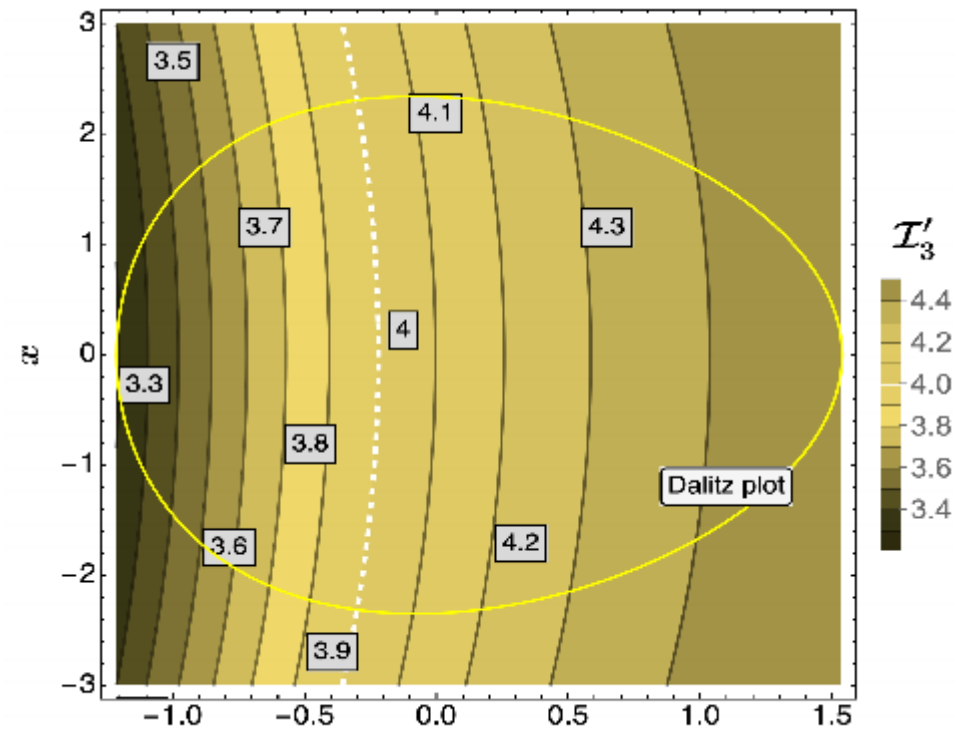
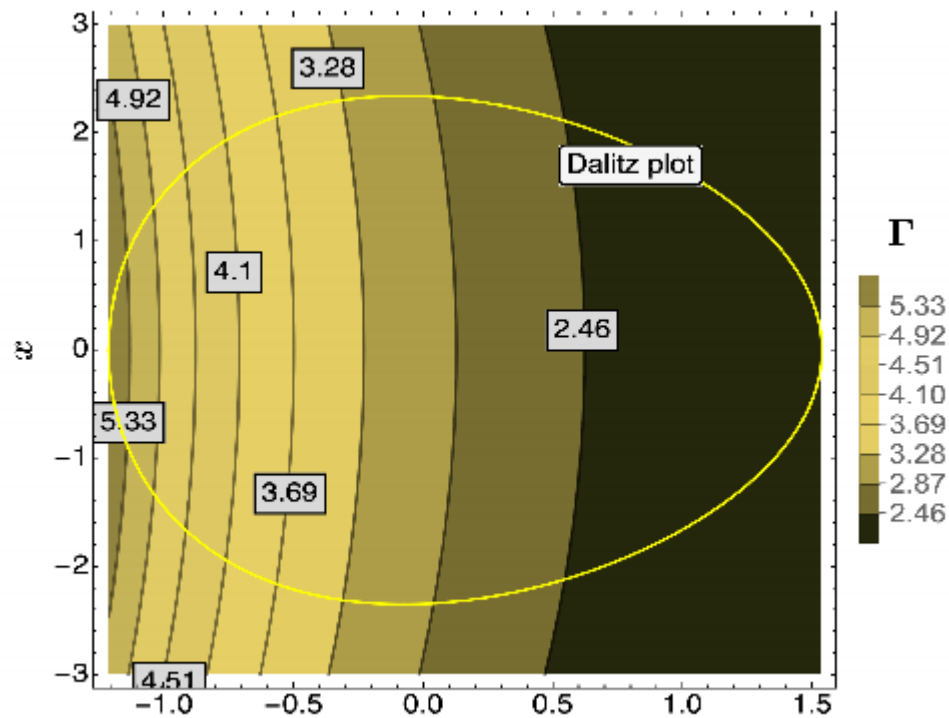
$$x = \frac{s_2 - s_1}{m_{\pi^+}^2} \quad y = \frac{s_3 - s_0}{m_{\pi^+}^2} \quad s_i = (p_j + p_k)^2$$

$$s_0 = (m_{K_L}^2 + m_{\pi^+}^2 + m_{\pi^-}^2 + m_{\pi^0}^2)/3$$

Values and relative uncertainties of α_i , ζ_i , β_i and ξ_i

taken from best fit of

J. Bijnens, P. Dhonte, and F. Borg, *K $\rightarrow$ 3 π decays in chiral perturbation theory*, Nucl. Phys. B **648** (2003) 317–344, [[hep-ph/0205341](#)].



$\Gamma = 2.600 \pm 0.008$ $\longrightarrow$ Inside allowed region of Dalitz plot
benchmark value

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Quantum-entangled pions*, *Phys. Rev. D* **112** (2025), no. 9 L091301, [arXiv:2506.05464].

- Entanglement is quantified and it is well **above 5σ level**

$$\mathcal{C}[\eta] = 0.979 \pm 0.001$$

$$F_3 = 0.959 \pm 0.002$$

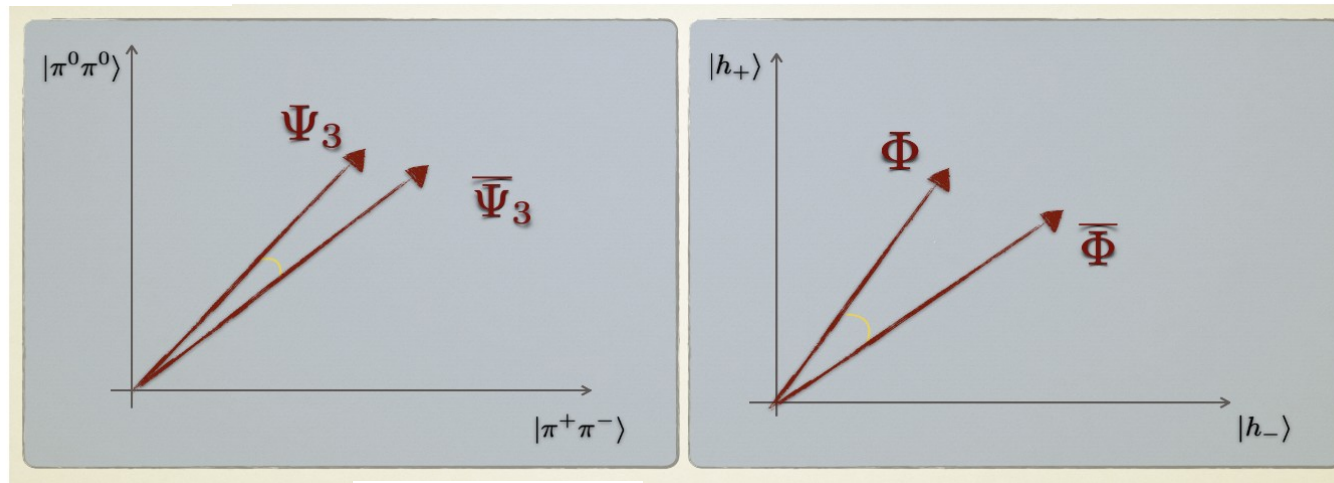
- Non-contextuality and Bell locality are all violated well **above 5σ level**

$$\text{CNT}\mathbb{X}\mathbb{T}_9 = 3.177 \pm 0.001$$

$$\mathcal{I}'_3 = 4.257 \pm 0.003$$

Measuring CP violations using Quantum state Tomography

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and
L. Marzola, *Measuring CP violation using quantum
state tomography*, [arXiv:2512.07939](https://arxiv.org/abs/2512.07939).



bar $\rightarrow$ CP-conjugated states

bi-partite neutral state in **isospin** ($|\pi^i \pi^j\rangle$) and **helicities** ($|h_{\pm}\rangle$)

● difference between a state and its CP-conjugated partner can be measured by

Trace distance

$$\mathcal{D}^T(\rho, \bar{\rho}) = \frac{1}{2} \text{Tr} \sqrt{(\rho - \bar{\rho})^\dagger (\rho - \bar{\rho})}$$

- Analyze direct CP violations in the K_L decay using the density matrices Trace distance
- Start with the quantum pion states coming from the decays of

$$|K^0\rangle \quad \longrightarrow \quad |\Psi_3\rangle = \xi_1 \left[|\psi_{\pi^+}(\vec{p}) \psi_{\pi^-}(-\vec{p})\rangle + |\psi_{\pi^-}(\vec{p}) \psi_{\pi^+}(-\vec{p})\rangle \right] + \xi_2 |\psi_{\pi^0}(\vec{p}) \psi_{\pi^0}(-\vec{p})\rangle$$

$$|\bar{K}^0\rangle \quad \longrightarrow \quad |\bar{\Psi}_3\rangle = \bar{\xi}_1 \left[|\psi_{\pi^+}(\vec{p}) \psi_{\pi^-}(-\vec{p})\rangle + |\psi_{\pi^-}(\vec{p}) \psi_{\pi^+}(-\vec{p})\rangle \right] + \bar{\xi}_2 |\psi_{\pi^0}(\vec{p}) \psi_{\pi^0}(-\vec{p})\rangle$$

corresponding density matrices

$$\rho_3 = \frac{|\Psi_3\rangle\langle\Psi_3|}{\langle\Psi_3|\Psi_3\rangle} \quad \text{and} \quad \bar{\rho}_3 = \frac{|\bar{\Psi}_3\rangle\langle\bar{\Psi}_3|}{\langle\bar{\Psi}_3|\bar{\Psi}_3\rangle}$$

using the basis: $\pi^+ = (1, 0, 0)^T$, $\pi^0 = (0, 1, 0)^T$ and $\pi^- = (0, 0, 1)^T$

$$\rho_3 = \frac{1}{2|\xi_1|^2 + |\xi_2|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & |\xi_1|^2 & 0 & \xi_1 \xi_2^* & 0 & |\xi_1|^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_2 \xi_1^* & 0 & |\xi_2|^2 & 0 & \xi_2 \xi_1^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & |\xi_1|^2 & 0 & \xi_1 \xi_2^* & 0 & |\xi_1|^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Quantum entangled pions redux

including CP-violations effects

From the decay of K_S $\Rightarrow |\gamma| = \frac{|\langle \pi^0 \pi^0 | \mathcal{L}_W | K_S \rangle|}{|\langle \pi^+ \pi^- | \mathcal{L}_W | K_S \rangle|} = 0.9349 \pm 0.0011$

$$\text{CNT}\mathbb{X}\mathbb{T}_9 = \frac{8|\gamma|^2 + 19}{3|\gamma|^2 + 6} = 3.015 \pm 0.001$$

$$\mathcal{I}_3 = \text{Tr} [\mathcal{B}\rho] = 2.894 \pm 0.001$$

$$\mathcal{E}[\rho] = \frac{1}{|\gamma|^2 + 2} \left[|\gamma|^2 \log \left(\frac{|\gamma|^2 + 2}{|\gamma|^2} \right) + 2 \log (|\gamma|^2 + 2) \right] = 1.097 \pm 0.001$$

From the decay of K_L $\Rightarrow |\gamma'| = \frac{|\langle \pi^0 \pi^0 | \mathcal{L}_W | K_L \rangle|}{|\langle \pi^+ \pi^- | \mathcal{L}_W | K_L \rangle|} = 0.930 \pm 0.002$

$$\text{CNT}\mathbb{X}\mathbb{T}_9 = 3.016 \pm 0.001 \quad \mathcal{I}_3 = 2.895 \pm 0.001 \quad \mathcal{E}[\rho] = 1.096 \pm 0.001$$

- also the two pions state from the K_L decay exhibits comparable quantum entanglement, as well as non-contextuality and Bell non-locality violations.

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | \mathcal{L}_W | K_L \rangle}{\langle \pi^0 \pi^0 | \mathcal{L}_W | K_S \rangle} \quad \text{and} \quad \eta_{+-} = \frac{\langle \pi^+ \pi^- | \mathcal{L}_W | K_L \rangle}{\langle \pi^+ \pi^- | \mathcal{L}_W | K_S \rangle}$$

$$\eta_{00} \equiv |\eta_{00}| e^{i\phi_{00}}$$

$$\eta_{+-} \equiv |\eta_{+-}| e^{i\phi_{+-}}$$

$$\xi_1 = (1 + \eta_{+-}) \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2}(1 + \varepsilon)} \langle \pi^+ \pi^- | \mathcal{L}_W | K_S \rangle, \quad \bar{\xi}_1 = (1 - \eta_{+-}) \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2}(1 - \varepsilon)} \langle \pi^+ \pi^- | \mathcal{L}_W | K_S \rangle,$$

$$\xi_2 = (1 + \eta_{00}) \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2}(1 + \varepsilon)} \langle \pi^0 \pi^0 | \mathcal{L}_W | K_S \rangle, \quad \bar{\xi}_2 = (1 - \eta_{00}) \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2}(1 - \varepsilon)} \langle \pi^0 \pi^0 | \mathcal{L}_W | K_S \rangle.$$

$$|\gamma| = \frac{|\langle \pi^0 \pi^0 | \mathcal{L}_W | K_S \rangle|}{|\langle \pi^+ \pi^- | \mathcal{L}_W | K_S \rangle|} \quad \gamma = (0.9357 \pm 0.0010) + i(0.0670 \pm 0.0010)$$

$$\varepsilon = (2.280 \pm 0.013) \times 10^{-3} e^{i\pi/4} \longrightarrow \text{Indirect CP violation parameter}$$

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Measuring CP violation using quantum state tomography*, [arXiv:2512.07939](https://arxiv.org/abs/2512.07939).

parameter	value
$ \eta_{00} $	$(2.220 \pm 0.011) \times 10^{-3}$
$ \eta_{+-} $	$(2.232 \pm 0.011) \times 10^{-3}$
ϕ_{00}	$(43.52 \pm 0.05)^\circ$
ϕ_{+-}	$(43.51 \pm 0.05)^\circ$
$ \eta_{00}/\eta_{+-} $	0.9950 ± 0.0007

Direct CP violation tested at a **significance** of about **7.2 σ**

$$\mathcal{D}^T(\rho_3, \bar{\rho}_3) \Big|_{\text{CP}} = (1.10 \pm 0.14) \times 10^{-5}$$

CP violation with SU(3) octet pions

$$\Pi = \begin{pmatrix} \eta/\sqrt{6} + \pi^0/\sqrt{2} & \pi^+/\sqrt{2} & K^+/\sqrt{2} \\ \pi^-/\sqrt{2} & \eta/\sqrt{6} - \pi^0/\sqrt{2} & K^0/\sqrt{2} \\ K^-/\sqrt{2} & \bar{K}^0/\sqrt{2} & -\sqrt{2/3}\eta \end{pmatrix}$$

Two pions states

$$\begin{aligned} |\Psi_8\rangle = & \xi_1 \left[|\psi_{\pi^+}(\vec{p}) \psi_{\pi^-}(-\vec{p})\rangle + |\psi_{\pi^-}(\vec{p}) \psi_{\pi^+}(-\vec{p})\rangle \right] + \xi_2 |\psi_{\pi^0}(\vec{p}) \psi_{\pi^0}(-\vec{p})\rangle + \xi_3 |\psi_{\eta}(\vec{p}) \psi_{\eta}(-\vec{p})\rangle \\ & + \xi_4 \left[|\psi_{K^+}(\vec{p}) \psi_{K^-}(-\vec{p})\rangle + |\psi_{K^-}(\vec{p}) \psi_{K^+}(-\vec{p})\rangle \right] + \xi_5 \left[|\psi_{K^0}(\vec{p}) \psi_{\bar{K}^0}(-\vec{p})\rangle + |\psi_{\bar{K}^0}(\vec{p}) \otimes \psi_{K^0}(-\vec{p})\rangle \right] \\ & + \xi_6 \left[|\psi_{K^+}(\vec{p}) \psi_{\pi^-}(-\vec{p})\rangle + |\psi_{\pi^-}(\vec{p}) \psi_{K^+}(-\vec{p})\rangle \right] + \xi_7 \left[|\psi_{K^-}(\vec{p}) \psi_{\pi^+}(-\vec{p})\rangle + |\psi_{\pi^+}(\vec{p}) \psi_{K^-}(-\vec{p})\rangle \right] \\ & + \xi_8 \left[|\psi_{K^0}(\vec{p}) \psi_{\pi^0}(-\vec{p})\rangle + |\psi_{\pi^0}(\vec{p}) \psi_{K^0}(-\vec{p})\rangle \right] + \xi_9 \left[|\psi_{\bar{K}^0}(\vec{p}) \psi_{\pi^0}(-\vec{p})\rangle + |\psi_{\pi^0}(\vec{p}) \psi_{\bar{K}^0}(-\vec{p})\rangle \right] \\ & + \xi_{10} \left[|\psi_{\pi^0}(\vec{p}) \psi_{\eta}(-\vec{p})\rangle + |\psi_{\eta}(\vec{p}) \psi_{\pi^0}(-\vec{p})\rangle \right] + \xi_{11} \left[|\psi_{\bar{K}^0}(\vec{p}) \psi_{\eta}(-\vec{p})\rangle + |\psi_{\eta}(\vec{p}) \psi_{\bar{K}^0}(-\vec{p})\rangle \right], \end{aligned}$$

Density matrices: $\rho_8 = \frac{|\Psi_8\rangle\langle\Psi_8|}{\langle\Psi_8|\Psi_8\rangle}$ and $\bar{\rho}_8 = \frac{|\bar{\Psi}_8\rangle\langle\bar{\Psi}_8|}{\langle\bar{\Psi}_8|\bar{\Psi}_8\rangle}$

From B meson decays

Decay	BR ($\times 10^{-5}$)	Asymmetry
$B^0 \rightarrow \pi^0 \pi^0$	0.155 ± 0.017	$C_{\pi^0 \pi^0} = -0.25 \pm 0.20$
$B^0 \rightarrow \pi^+ \pi^-$	0.537 ± 0.020	$C_{\pi^+ \pi^-} = -0.314 \pm 0.030^*$
$B^0 \rightarrow K_S^0 K_S^0$	0.121 ± 0.016	$C_{K_S^0 K_S^0} = 0.0 \pm 0.4$
$B^0 \rightarrow K^+ K^-$	0.0078 ± 0.0015	—
$B^0 \rightarrow \pi^0 K_S^0$	1.01 ± 0.04	$C_{\pi^0 K_S^0} = 0.00 \pm 0.08$
$B^0 \rightarrow \pi^- K^+$	2.00 ± 0.04	$A_{CP}(K^- \pi^+) = -0.0831 \pm 0.0031^*$
$B^0 \rightarrow \pi^0 \eta$	0.041 ± 0.017	—
$B^0 \rightarrow K_S^0 \eta$	0.123 ± 0.027	—
$B^0 \rightarrow \eta \eta$	< 0.1	—

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Measuring CP violation using quantum state tomography*, [arXiv:2512.07939](https://arxiv.org/abs/2512.07939).

$$\mathcal{E}[\rho_8] = 1.670 \pm 0.008, \quad \mathcal{E}[\bar{\rho}_8] = 1.667 \pm 0.008$$

$$\mathcal{D}^T(\rho_8, \bar{\rho}_8) \Big|_{\text{CP}} = 0.11 \pm 0.02$$

Trace distance does not vanish with at a **significance** of about **5.5σ**

From D meson decay

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Measuring CP violation using quantum state tomography*, [arXiv:2512.07939](https://arxiv.org/abs/2512.07939).

Decay	BR ($\times 10^{-2}$)	Asymmetry
$D^0 \rightarrow \pi^0 \pi^0$	0.0826 ± 0.0025	$A_{CP}(\pi^0 \pi^0) = 0.000 \pm 0.006$
$D^0 \rightarrow \pi^+ \pi^-$	0.1453 ± 0.0024	$A_{CP}(\pi^+ \pi^-) = 0.0013 \pm 0.0014$
$D^0 \rightarrow K_S^0 K_S^0$	0.0141 ± 0.0005	$A_{CP}(K_S^0 K_S^0) = -0.011 \pm 0.019$
$D^0 \rightarrow K^+ K^-$	0.408 ± 0.006	$A_{CP}(K^+ K^-) = (4 \pm 5) \times 10^{-4}$
$D^0 \rightarrow \pi^0 K_S^0$	1.240 ± 0.022	$A_{CP}(\pi^0 K_S^0) = -0.0020 \pm 0.0017$
$D^0 \rightarrow \pi^+ K^-$	3.945 ± 0.03	$A_{CP}(\pi^+ K^-) = 0.002 \pm 0.005$
$D^0 \rightarrow \pi^- K^+$	0.0150 ± 0.0007	$A_{CP}(\pi^- K^+) = -0.009 \pm 0.014$
$D^0 \rightarrow K_S^0 \eta$	0.434 ± 0.016	$A_{CP}(K_S^0 \eta) = 0.005 \pm 0.005$
$D^0 \rightarrow \eta \eta$	0.211 ± 0.019	–

$$\mathcal{D}^T(\rho_8, \bar{\rho}_8) \Big|_{\text{CP}} = (3.31 \pm 1.02) \times 10^{-3},$$

Trace distance does not vanish with at a **significance** of about **3.3σ**

Conclusions

- As for the spin, also the Flavor ($SU(2)$ or $SU(3)$) can be entangled
- we considered bi- and tri-partite systems of pions from the neutral Kaon decays
- quantum state tomography (QST) used to reconstruct the Flavor quantum state
- we found that these states are entangled, non-contextual and Bell non-locals
- QST used also to test CP violations in Kaon and B meson decays in two pions
- The trace distance for the density matrices of quantum states and their CP-conjugated ones offers alternative powerful method to test direct CP violations.
- We show that direct CP violation in two pions systems can be tested with 7.2σ significance from neutral Kaon decays
- We have also explored D and B meson decays in two pions states from $SU(3)$ pions octet, and tested direct CP violations, although with much less significance than in the Kaon sector.